MAT 121 §3.5 II #s 71, 74, 77, 89, 92

#s 69-80 Find the intercepts & graph each polynomial function.

#71 See 3.5 I. I already did this one.

74 $f(x) = x^4 - 3x^2 - 4$ is QUADRATIC IN FORM

$u = x^2 \Longrightarrow u^2 - 3u - 4$. $\qquad f(0) = -4 \rightsquigarrow (0,-4)$

zeros: $(u-4)(u+1) = 0$

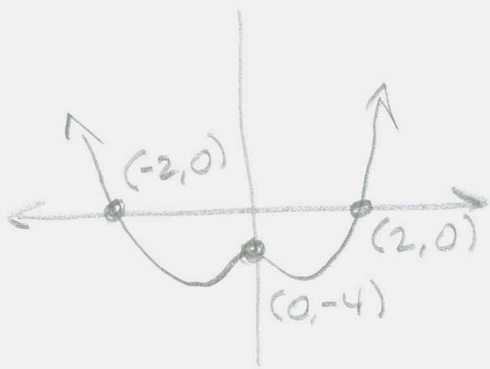
$\Longrightarrow u = 4$ OR $u = -1$

$x^2 = 4$ $\qquad x^2 = -1$

$\boxed{x = \pm 2}$ $\qquad$ No Real Soln

$(-2,0)$, $(2,0)$

$f(-x) = f(x)$ is EVEN!



77 $f(x) = x^4 + \underbrace{x^3 - 3x^2}_{1} - \underbrace{x + 2}_{2}$ No Easy tricks.

2 or 0 positive real zeros

$f(-x) = \underbrace{x^4 - x^3}_{1} - \underbrace{3x^2 + x}_{2} + 2$

2 or 0 negative real zeros.

Rational zeros:

$a_0 = 2$ $\qquad \pm \dfrac{\text{FACTORS OF } a_0}{\text{FACTORS OF } a_n} = \pm 1, \pm 2$

$a_n = 1$

MAT 121 §3.5 II #s 74, 77, 89, 92

$\underline{1}\rfloor$	1	1	−3	−1	2
		1	2	−1	−2
	1	2	−1	−2	0 Yes!
	x^3	x^2	x^1	x^0	r

$(x-1)(x^3+2x-x-2)$

Now: x^3+2x^2-x-2 is what we're left with.

It factors by grouping:

$x^2(x+2)-1(x+2)$

$= (x+2)(x^2-1)$

$= (x+2)(x-1)(x+1)$

$x=-2$ cross

$x=-1$ cross

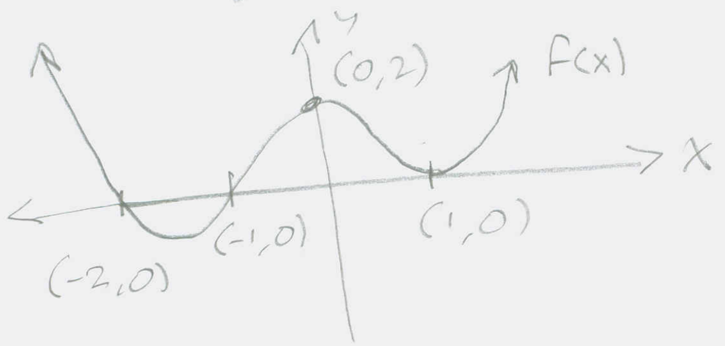
$x=1$ touch

So, $f(x) = (x-1)(x+2)(x-1)(x+1)$

$= (x-1)^2(x+1)(x+2)$

End Behavior: EB for $x^4+x^3-3x^2-x+2$ is $y=x^4$ ↖ … ↗

$f(0)=2 \Rightarrow (0,2)$



Without a grapher, we're not sure exactly where or what value the turning points are between $x=-2$, $x=-1$ and between $x=-1$, $x=1$. We just know they exist.

MAT 121 §3.5 II #s 89, 92

#s 89–94 are Intermediate Value Theorem questions – Easy if you know IVT, Hard if you don't. Show each polynomial function has a zero in the given interval.

89 $f(x) = 8x^4 - 2x^2 + 5x - 1$; $[0, 1]$

$f(0)$:

$$\begin{array}{r|rrrr} 0 & 8 & -2 & +5 & -1 \\ & & 0 & 0 & 0 \\ \hline & 8 & -2 & 5 & -1 = f(0) \end{array}$$

$-1 = f(0)$, which I could've found by inspection!

$f(1)$:

$$\begin{array}{r|rrrr} 1 & 8 & -2 & 5 & -1 \\ & & 1 & -1 & 4 \\ \hline & 8 & -1 & 4 & 3 = f(1) \end{array}$$

$\boxed{\text{Since } f(0) = -1 < 0 < 3 = f(1) \text{, we know there is } c \in (0,1) \text{ such that } f(c) = 0 \text{, by I.V.T.}}$

92 $f(x) = 3x^3 - 10x + 9$; $[-3, -2]$

$$\begin{array}{r|rrrr} -3 & 3 & 0 & -10 & 9 \\ & & -9 & 27 & -51 \\ \hline & 3 & -9 & 17 & -42 = f(-3) \end{array}$$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -10 & 9 \\ & & -6 & 12 & -4 \\ \hline & 3 & -6 & 2 & 5 = f(-2) \end{array}$$

$\boxed{\text{Since } f(-3) = -42 < 0 < 5 = f(-2), \ \exists\, c \in (-3,-2) \ni f(c) = 0 \text{, by IVT}}$